Exercise 1: Prove that when,

$$E_1 = E_2 \text{ and } G_{12} = \frac{E_1}{2(1+\nu_{12})}$$

the material is isotropic.

Hind: Use the first relation in (D.36) and show that the modulus is independent of orientation.

Solution:

Consider the first relation (D.36),

$$\frac{1}{E_x} = \frac{1}{E_1}c^4 + \frac{1}{E_2}s^4 - 2\frac{\nu_{12}}{E_1}c^2s^2 + \frac{1}{G_{12}}c^2s^2$$

Set $E_1 = E_2$ and $G_{12} = \frac{E_1}{2(1+\nu_{12})}$.

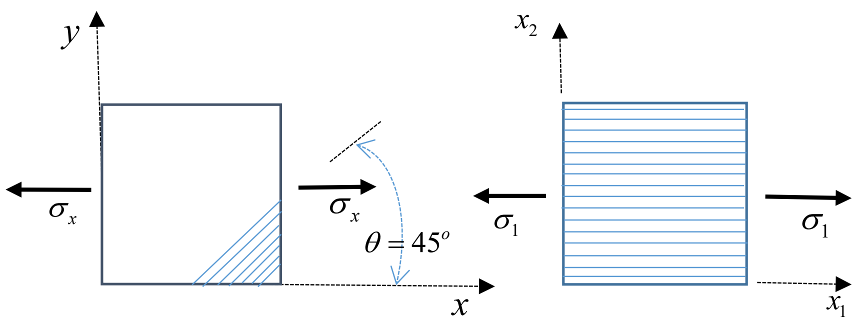
$$\frac{1}{E_x} = \frac{1}{E_1}c^4 + \frac{1}{E_1}s^4 - 2\frac{\nu_{12}}{E_1}c^2s^2 + \frac{2(1+\nu_{12})}{E_1}c^2s^2$$

$$\frac{1}{E_x} = \frac{1}{E_1}\left(c^4 + s^4\right) - 2\frac{\nu_{12}}{E_1}c^2s^2 + \frac{2}{E_1}c^2s^2 + \frac{2\nu_{12}}{E_1}c^2s^2 = \frac{1}{E_1}\left(c^4 + s^4 + 2c^2s^2\right) = \frac{1}{E_1}\left(c^2 + s^2\right)^2$$

With $c = \cos\theta$, $s = \sin\theta$, and $\left(c^2 + s^2\right) = 1$, we get $E_x = E_1$.

We can repeat the same steps with E_2 to get $E_x = E_2$ which proves that the material is isotropic.

Exercise 2: Two specimens of an orthotropic material (unidirectional laminate) are subjected to uniaxial traction, as shown in the figure, with $\sigma_x = \sigma_1 = \sigma$. Calculate the resulting lateral strain.



The material properties are,

$E_1 = 145$ GPa,
$E_2 = 10.4$ GPa,
$G_{12} = 7.6$ GPa,
$\nu_{12} = 0.28$

Solution:

For the 45° case (see Figure D7), use (D.33) and (D.24) to obtain the transversal strain,

$$\varepsilon_y = -\frac{\nu_{xy}}{E_x}\sigma_x = S_{xy}\sigma_x$$

Next take S_{xy} from (D.31) with $c = s = \sqrt{2}/2$ and (D.21b) to relate,

$$S_{11} = \frac{1}{E_1}, S_{22} = \frac{1}{E_2}, S_{12} = -\frac{\nu_{12}}{E_1}, S_{66} = \frac{1}{G_{12}}$$

$$\Rightarrow S_{xy} = \left(S_{11} + S_{22} - S_{66}\right)c^2 s^2 + S_{12}\left(c^4 + s^4\right) = 8.10\text{x}10^{-12}(\text{GPa})^{-1}$$

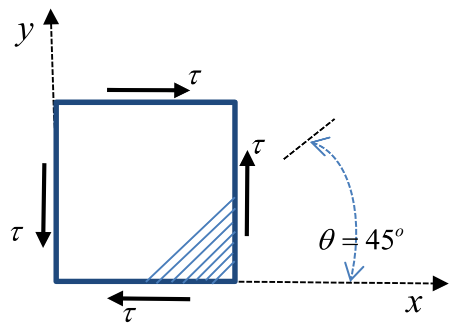
$$\Rightarrow \varepsilon_y = -\frac{\nu_{xy}}{E_x}\sigma_x = S_{xy}\sigma_x = -8.10\text{x}10^{-12}\sigma$$

For 0° we have from (D.21),

$$\Rightarrow \varepsilon_2 = -\frac{\nu_{12}}{E_1}\sigma = -1.93\text{x}10^{-12}\sigma$$

Thus, the lateral deformation is larger (absolute value) in the 45° case.

Exercise 3: A uniaxial lamina is loaded in pure shear $\sigma_{xy} = \tau$ at 45° with the principal material system axes. Express the resulting strain components in terms of the orthotropic elastic constants $E_1, E_2, G_{12}, \nu_{12}\,(\text{or}\,\nu_{21})$.



Solution:

We use constitutive relation (D.33), with $\sigma_x = \sigma_y = 0,\ \ \sigma_{xy} = \tau$,

$$\begin{pmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{pmatrix} = \begin{pmatrix} \frac{1}{E_x} & -\frac{\nu_{yx}}{E_y} & \frac{\eta_{sx}}{G_{xy}} \\ -\frac{\nu_{xy}}{E_x} & \frac{1}{E_y} & \frac{\eta_{sy}}{G_{xy}} \\ \frac{\eta_{xs}}{E_x} & \frac{\eta_{ys}}{E_y} & \frac{1}{G_{xy}} \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ \tau \end{pmatrix} \quad \text{or} \quad \begin{aligned} \varepsilon_x &= \frac{\eta_{sx}}{G_{xy}}\tau = S_{sx}\tau \ \ (S_{sx} = S_{xs}) \\ \varepsilon_y &= \frac{\eta_{sy}}{G_{xy}}\tau = S_{sy}\tau \ \ (S_{sy} = S_{ys}) \\ \gamma_{xy} &= \frac{1}{G_{xy}}\tau = S_{ss}\tau \ \ (S_{sy} = S_{ys}) \end{aligned}$$

Taking S_{xs}, S_{ys}, S_{ss} from (D.31) at 45° and (D.21b) we obtain,

$$\varepsilon_x = \frac{\eta_{sx}}{G_{xy}}\tau = \frac{\tau}{2}\left(\frac{1}{E_1} - \frac{1}{E_2}\right)$$

$$\varepsilon_y = \frac{\eta_{sy}}{G_{xy}}\tau = \frac{\tau}{2}\left(\frac{1}{E_1} - \frac{1}{E_2}\right)$$

$$\gamma_{xy} = \frac{1}{G_{xy}}\tau = \tau\left(\frac{1+\nu_{12}}{E_1} + \frac{1+\nu_{21}}{E_2}\right)$$